Statistical entropy of quantum systems

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Understanding the relationship between von Neumann (VN) entropy and thermodynamic (TH) entropy is a fundamental challenge in quantum statistical mechanics. For a quantum system in a random pure state, the average VN entropy of the subsystem scales as $\ln(D_1)$, where D_1 is the subsystem's Hilbert space dimension[1, 2]. We can strengthen this result for thermalized systems. The VN entropy of the subsystem, when averaged over the subspace $(\mathscr{H}_E)$ corresponding to the narrow energy shell $(E, E+\Delta E)$ of the full system, scales as $\ln\left(\widetilde{d}_1\right)$. Here, $\widetilde{d}_1 = D_1^{\gamma}$ represents the effective Hilbert space dimension of the subsystem relevant to $\mathscr{H}_E$, with $\gamma = \ln(d_E)/\ln(D)$, D as the total Hilbert space dimension and d_E as the dimension of $\mathscr{H}_E$. This result demonstrates that the VN entropy of a subsystem in thermal equilibrium is equivalent to its TH entropy, providing a statistical interpretation of TH entropy in the subsystem of quantum systems. Numerical results from a one-dimensional spin-1/2 chain with next-nearest neighbor interactions confirm this scaling, bridging quantum statistical mechanics and thermodynamics [3].

References

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